This print-out should have 11 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

**Hinged Mirrors 01**

**001 10.0 points**

The reflecting surfaces of two intersecting flat mirrors are at an angle of $55^\circ$, as shown in the figure. A light ray strikes the horizontal mirror at an angle of $53^\circ$ with respect to the mirror’s surface.

![Figure is not drawn to scale.](image)

Calculate the angle $\phi$.

Correct answer: $70^\circ$.

**Explanation:**

**Basic Concept:**

$$\theta_{\text{incident}} = \theta_{\text{reflected}}$$

**Solution:**

![Figure is to scale.](image)

The sum of the angles in a triangle is $180^\circ$.

In the triangle on the left we have angles

$$\theta, \quad \frac{180^\circ - \theta_1}{2}, \quad \text{and} \quad \frac{180^\circ - \theta_2}{2}, \quad \text{so}$$

$$180^\circ = \theta + \frac{180^\circ - \theta_1}{2} + \frac{180^\circ - \theta_2}{2}, \quad \text{or}$$

$$\theta_1 + \theta_2 = 2 \theta. \quad (1)$$

In the triangle on the right we have angles

$$\theta_1, \quad \theta_2, \quad \text{and} \quad \phi.$$  

$$180^\circ = \theta_1 + \theta_2 + \phi, \quad \text{so}$$

$$\theta_1 + \theta_2 = 180^\circ - \phi. \quad (2)$$

Combining Eq. 1 and 2, we have

$$\phi = 180^\circ - 2 \theta$$

$$= 180^\circ - 2(55^\circ)$$

$$= 70^\circ.$$  

As a matter of interest, in the upper-half of the figure the angles (clockwise) in the triangles from left to right are

$$37^\circ, \quad 37^\circ, \quad \text{and} \quad 106^\circ;$$

$$74^\circ, \quad 35^\circ, \quad \text{and} \quad 71^\circ;$$

$$109^\circ, \quad 18^\circ, \quad \text{and} \quad 53^\circ;$$

$$127^\circ, \quad 18^\circ, \quad \text{and} \quad 35^\circ;$$

and in the lower-half of the figure the angles (counter-clockwise) in the triangles from left to right are

$$18^\circ, \quad 18^\circ, \quad \text{and} \quad 144^\circ;$$

$$36^\circ, \quad 35^\circ, \quad \text{and} \quad 109^\circ;$$

$$71^\circ, \quad 37^\circ, \quad \text{and} \quad 72^\circ;$$

$$108^\circ, \quad 37^\circ, \quad \text{and} \quad 35^\circ.$$

**Perpendicular Mirrors**

**002 (part 1 of 2) 10.0 points**

Consider the case in which light ray A is incident on mirror 1, as shown in the figure. The reflected ray is incident on mirror 2 and subsequently reflected as ray B. Let the angle of incidence (with respect to the normal) on mirror 1 be $\theta_A = 58^\circ$ and the point of incidence be located 20 cm from the edge of contact between the two mirrors.
What is the angle of the reflection of ray B (with respect to the normal) on mirror 2?

Correct answer: 32°.

Explanation:
If the angle of the incident ray A is \( \theta_A \), the angle of reflection must also be \( \theta_A \). Since the mirrors are perpendicular to each other, angle \( \theta_B \) is equal to \( 90^\circ - \theta_A \)

\[
\theta_B = 90^\circ - \theta_A \\
= 90^\circ - 58^\circ \\
= 32^\circ .
\]

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003 (part 2 of 2) 10.0 points

Determine the angle between the rays A and B.

1. 116°
2. 0°
3. 180° correct
4. 90°
5. 64°
6. 32°
7. 58°

**Explanation:**
Refer to the figure below. Consider the angle each ray makes with mirror 1. Ray A makes an angle of \( \phi_A \) with mirror 1. This is given by

\[
\phi_A = 90^\circ - \theta_A = \theta_B .
\]

The normal to mirror 2 is parallel to mirror 1. Thus the angle ray B makes with mirror 1 (assumed to be extended) is \( \theta_B \). Both rays make the same angle with respect to mirror 1. They are parallel, but point in the opposite directions. Therefore the angle between them is 180°.

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**004 10.0 points**

In the figure, a ray of light enters the liquid from air and is bent toward the normal as shown.

What is the index of refraction \( n \) for the liquid?

Correct answer: 1.25.

**Explanation:**
Let: \( a = 10 \), and \( b = 8 \).

By Snell’s Law,

\[
\frac{n_1 \sin \theta_1}{n_2} = \frac{n_2 \sin \theta_2}{c}
\]

\[
n_2 = \frac{a}{c} \cdot \frac{c}{b} = \frac{a}{b} = \frac{10}{8} = 1.25.
\]

**Explanation:**

Given: \( \theta_{in} = 27.3^\circ \),

\( \theta_{12} = 31.4^\circ \), and

\( \theta_{32} = 36.4^\circ \).

Applying Snell’s Law with sheet 1 on top of sheet 2,

\[
n_1 \sin \theta_{in} = n_2 \sin \theta_{12}
\]

\[
n_2 = \frac{n_1 \sin \theta_{in}}{\sin \theta_{12}}.
\]

Applying Snell’s Law with sheet 3 on top of sheet 2,

\[
n_3 \sin \theta_{in} = n_2 \sin \theta_{32}
\]

\[
n_3 = \frac{n_1 \sin \theta_{32}}{\sin \theta_{12}}.
\]

Applying Snell’s law with sheet 1 on top of sheet 3,

\[
n_1 \sin \theta_{in} = n_3 \sin \theta_{13}
\]

\[
\sin \theta_{13} = \frac{n_1 \sin \theta_{in}}{n_3}
\]

\[
= \left( \frac{\sin \theta_{12}}{\sin \theta_{32}} \right)
\]

\[
= \left( \frac{\sin 27.3^\circ}{\sin 36.4^\circ} \right)
\]

\[
= 0.402685
\]

and \( \theta_{13} = 23.7461^\circ \).

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**Spherical Mirror A 02**

006 (part 1 of 2) 10.0 points

A convex spherical mirror has a radius of curvature of 26.6 cm. The object distance is 12.4 cm.

Three sheets of plastic have unknown indices of refraction. Sheet 1 is placed on top of sheet 2, and a laser beam is directed onto the sheets from above so that it strikes the interface at an angle of 27.3° with the normal, and refracts in sheet 2 at an angle of 31.4° with the normal. The experiment is repeated with sheet 3 on top of sheet 2 and, with the same angle of incidence, the refracted beam makes an angle of 36.4° with the normal.

If the experiment is repeated again with sheet 1 on top of sheet 3, what is the expected angle of refraction in sheet 3? Assume the same angle of incidence.

Correct answer: 23.7461°.
Find the magnitude of the image distance.

Correct answer: 6.41712 cm.

**Explanation:**

\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R} \quad M = \frac{h'}{h} = -\frac{q}{p}
\]

**Convex Mirror** \( f < 0 \)

\( \infty > p > 0 \) \( f < q < 0 \) \( 0 < M < 1 \)

**Note:** The radius of curvature for a convex mirror is negative, \( R = -26.6 \).

**Solution:** Substituting these values into the mirror equation

\[
q = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{2}{R} - \frac{1}{p}} = \frac{1}{\frac{2}{-26.6} - \frac{1}{12.4 \text{ cm}}} = \frac{1}{-6.41712 \text{ cm}}.
\]

\[|q| = \left|\frac{6.41712 \text{ cm}}{}\right|.
\]

\[007 \text{ (part 2 of 2) 10.0 points}

Find the magnification.

Correct answer: 0.51751.

**Explanation:**

\[
M = -\frac{q}{p} = -\frac{(-6.41712 \text{ cm})}{(12.4 \text{ cm})} = 0.51751.
\]

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**Spherical Mirror B 01 008 10.0 points**

A concave spherical mirror forms a real image 1.36 times the size of the object. The object distance is 26.1 cm.

Find the magnitude of the radius of curvature of the mirror.

Correct answer: 30.0814 cm.

**Explanation:**

\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R} \quad M = \frac{h'}{h} = -\frac{q}{p}
\]

**Concave Mirror** \( f > 0 \)

\( \infty > p > f \) \( f < q < \infty \) \( 0 > M > -\infty \)

**Note:** The radius of curvature for a concave mirror is positive.

\[M = -\frac{q}{p} = -1.36. \quad (1)
\]

Solving for \( q \), we have

\[q = -M p \quad (2)
\]

\[= -(-1.36) (26.1 \text{ cm}) = 35.496 \text{ cm}.\]
Substituting these values into the mirror equation

\[ R = 2 \frac{f}{p + q} = 2 \frac{1}{\frac{1}{p} + \frac{1}{q}} \] (3)

\[ = \frac{2}{\frac{1}{(26.1 \text{ cm})} + \frac{1}{(35.496 \text{ cm})}} = 30.0814 \text{ cm}. \]

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**Lens A 01**

**009 (part 1 of 2) 10.0 points**

A convergent lens has a focal length of 10.9 cm. The object distance is 36.1 cm.

Correct answer: 15.6147 cm.

Explanation:

\[ M = \frac{h'}{h} = \frac{-q}{p} = \frac{-(15.6147 \text{ cm})}{36.1 \text{ cm}} = -0.43254. \]

**Lens B 02**

**011 10.0 points**

A divergent lens forms a virtual image 0.41 times the size of the object. The object distance is 25.4 cm.

Correct answer: 17.6508 cm.

Explanation:
Note: The focal length for a divergent lens is negative.

\[ M = - \frac{q}{p} = 0.41. \]  

(1)

Solving for \( q \), we have

\[ q = - M p \]  

(2)

\[ = - (0.41) (25.4 \text{ cm}) \]

\[ = -10.414 \text{ cm}. \]

Substituting these values into the mirror equation

\[ f = \frac{1}{\frac{1}{p} + \frac{1}{q}} \]  

(3)

\[ = \frac{1}{\frac{1}{(25.4 \text{ cm})} + \frac{1}{(-10.414 \text{ cm})}} \]

\[ = -17.6508 \text{ cm}, \text{ so} \]

\[ |f| = 17.6508 \text{ cm}. \]