Temperature and Sound Waves
001 10.0 points
A sound wave has a frequency of 752 Hz in air and a wavelength of 0.47 m.
What is the temperature of the air? Assume the velocity of sound at 0°C is 330 m/s.

\[
\frac{1}{\sqrt{1 + x}} \approx \sqrt{1 - x} \quad \text{for} \quad x \ll 1.
\]

Correct answer: 40.1599°C.

Explanation:

Let:

\[
f = 752 \, \text{Hz}, \quad \lambda = 0.47 \, \text{m}, \quad \text{and} \quad v_0 = 330 \, \text{m/s}.
\]

The velocity of sound is

\[
v \approx \sqrt{\frac{B}{\rho}}.
\]

Since the density decreases as the temperature increases, the speed will increase with the square root of the temperature.

Assuming a linear dependence, we have

\[
\frac{\rho}{\rho_0} = 1 - \frac{T}{T_0}
\]

\[
\frac{v}{v_0} = \sqrt{\frac{\rho_0}{\rho}} = \sqrt{\frac{\rho_0}{\rho_0 \left(1 - \frac{T}{T_0}\right)}}
\]

\[
\approx \sqrt{1 + \frac{T}{T_0}}
\]

\[
T = T_0 \left[\left(\frac{v}{v_0}\right)^2 - 1\right]
\]

\[
= (273°C) \left[\left(\frac{353.44 \, \text{m/s}}{330 \, \text{m/s}}\right)^2 - 1\right]
\]

\[
= 40.1599°C
\]

Car Passing Stationary Siren
002 10.0 points
A car, moving at 94 mi/hr, passes a stationary police car whose siren has a frequency of 500 Hz.

What is the frequency change heard by an observer in the moving car as he passes the police car? The velocity of sound in air is 343 m/s. (1 mi = 1.609 km)

Correct answer: 122.486 Hz.

Explanation:

Let:

\[
v_o = 94 \, \text{mi/h}, \quad v_{sound} = 343 \, \text{m/s}, \quad \text{and} \quad f = 500 \, \text{Hz}.
\]

When the observer is approaching the police car, he hears a frequency of

\[
f_b = f \left(1 + \frac{v_o}{v_{sound}}\right)
\]

and while he is moving away from the police car, he hears a frequency of

\[
f_a = f \left(1 - \frac{v_o}{v_{sound}}\right),
\]

so the change of the frequency he hears is

\[
\Delta f = |f_a - f_b| = \frac{2 f v_o}{v_{sound}}
\]

\[
= 2 \left(500 \, \text{Hz}\right) \left(94 \, \text{mi/h}\right)
\]

\[
\times \frac{343 \, \text{m/s}}{1 \, \text{mi}} \times \frac{1000 \, \text{m}}{1 \, \text{km}} \times \frac{1 \, \text{h}}{3600 \, \text{s}}
\]

\[
= 122.486 \, \text{s}^{-1}.
\]
Closed Air Column

003 10.0 points
If the speed of sound in air is 340 m/s, what is approximately the length of a shortest air column closed at one end that will respond to a tuning fork of frequency 182 Hz?

Correct answer: 46.7033 cm.

Explanation:

Let : \( v = 340 \text{ m/s} \) and \( f = 182 \text{ Hz} \).

The length of the air column is

\[
L = \frac{n \lambda}{4} \quad (n = 1, 3, 5, \ldots).
\]

Since \( v = f \lambda \), the shortest air column has length

\[
L = \frac{v}{4f} = \frac{340 \text{ m/s} \times 100 \text{ cm}}{4(182 \text{ Hz}) \times 1 \text{ m}} = 46.7033 \text{ cm}.
\]

Open Organ Pipe

005 10.0 points
The fundamental frequency of an organ pipe open on both ends corresponds to sound whose frequency is 270.4 Hz. What is the length of the pipe? The speed of sound in air is 344 m/s.

Correct answer: 0.636095 m.

Explanation:

Let : \( v_s = 344 \text{ m/s} \) and \( f = 270.4 \text{ Hz} \).

The frequency of the \( n \text{th} \) harmonic in an air column with both ends open is given by

\[
f_n = n \frac{v_s}{2L}, \quad n = 1, 2, 3, \ldots.
\]

\( n = 1 \) gives the fundamental frequency, so

\[
f_1 = \frac{v_s}{2f} = \frac{344 \text{ m/s}}{2(270.4 \text{ Hz})} = 0.636095 \text{ m}.
\]

Human Ear Canal

004 10.0 points
The human ear canal is about 2.78 cm long. The canal is regarded as a tube open at one end and closed at the eardrum.

What is the fundamental frequency around which we would expect hearing to be best? The velocity of sound is 343 m/s.

Correct answer: 3084.53 Hz.

Explanation:

Let : \( L = 2.78 \text{ cm} \) and \( v = 343 \text{ m/s} \).

Standing Waves

006 (part 1 of 3) 10.0 points
An open vertical tube has water in it. A tuning fork vibrates over its mouth. As the water level is lowered in the tube, the fourth resonance is heard when the water level is 171.5 cm below the top of the tube.
What is the wavelength of the sound wave?
The speed of sound in air is $343 \text{ m/s}$.

Correct answer: 98 cm.

**Explanation:**

Let: $\ell = 171.5 \text{ cm}$ and $N = 4$.

When one end of a pipe is closed and the other end open, the wavelength is

$$\lambda = \frac{4\ell}{2N - 1}, \quad N = 1, 2, 3, \cdots,$$

since there is node at one end, so

$$\lambda = \frac{4\ell}{2N - 1} = \frac{4\ell}{2(4) - 1} = \frac{4(171.5 \text{ cm})}{2(4) - 1} = 98 \text{ cm}.$$

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**007 (part 2 of 3) 10.0 points**

What is the frequency of the sound wave; i.e., the tuning fork?

Correct answer: 350 s$^{-1}$.

**Explanation:**

Let: $v = 343 \text{ m/s}$.

The frequency is

$$f = \frac{v}{\lambda} = \frac{343 \text{ m/s}}{98 \text{ cm} \times \frac{100 \text{ cm}}{1 \text{ m}}} = 350 \text{ Hz}.$$

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**008 (part 3 of 3) 10.0 points**

The water continues to leak out the bottom of the tube.

When the tube next resonates with the tuning fork, what is the length of the air column?

Correct answer: 220.5 cm.

**Explanation:**

The next resonance will occur when the open vertical tube has a length $\frac{\lambda}{2}$ greater than its initial length:

$$\ell' = \ell + \frac{\lambda}{2} = 171.5 \text{ cm} + \frac{98 \text{ cm}}{2} = 220.5 \text{ cm}.$$