Electrical Attraction 03

001 10.0 points

Three charges are arranged in the \((x, y)\) plane as shown.

What is the magnitude of the resulting force on the 6 nC charge at the origin? The Coulomb constant is \(8.98755 \times 10^9\) N \(\cdot\) C^2/m^2.

Your answer must be within ±1.0%

Correct answer: 7.20113 nN.

Explanation:

Let:

\[ q_0 = 6 \times 10^{-9} \text{ C}, \]
\[ q_a = -6 \times 10^{-9} \text{ C}, \]
\[ r_a = 9 \text{ m}, \]
\[ q_b = 9 \times 10^{-9} \text{ C}, \]
\[ r_b = 9 \text{ m}, \]
\[ k_e = 8.98755 \times 10^9 \text{ N} \cdot \text{C}^2/\text{m}^2. \]

Applying Coulomb’s Law for \(q_0\) and \(q_a\),

\[
F_{oa} = -k_e \frac{q_0 q_a}{r_{oa}^2}
\]
\[
= -(8.98755 \times 10^9 \text{ N} \cdot \text{C}^2/\text{m}^2)
\times \frac{(6 \times 10^{-9} \text{ C})(-6 \times 10^{-9} \text{ C})}{(9 \text{ m})^2}
\]
\[
= 3.99447 \times 10^{-9} \text{ N}
\]
directed along the \(x\)-axis.

Applying Coulomb’s Law for \(q_0\) and \(q_b\),

\[
F_{ob} = -k_e \frac{q_0 q_b}{r_{ob}^2}
\]
\[
= -(8.98755 \times 10^9 \text{ N} \cdot \text{C}^2/\text{m}^2)
\times \frac{(6 \times 10^{-9} \text{ C})(9 \times 10^{-9} \text{ C})}{(9 \text{ m})^2}
\]
\[
= -5.9917 \times 10^{-9} \text{ N}
\]
directed along the \(y\)-axis.

The magnitude of the resultant force is

\[
\| \vec{F} \| = \sqrt{F_x^2 + F_y^2}
\]
\[
= \left[ (3.99447 \times 10^{-9} \text{ N})^2 \right. \\
+ \left. ((-5.9917 \times 10^{-9} \text{ N})^2 \right]^{1/2}
\]
\[
\times \frac{1 \times 10^9 \text{ nN}}{1 \text{ N}}
\]
\[
= 7.20113 \text{ nN}.
\]

Three Point Charges 02

002 10.0 points

Three equal charges 5 \(\mu\)C are located in the \(xy\)-plane, one at \((0 \text{ m}, 49 \text{ m})\), another at \((27 \text{ m}, 0 \text{ m})\), and the third at \((29 \text{ m}, -66 \text{ m})\).

Find the magnitude of the electric field at the origin due to these three charges. The value of Coulomb’s constant is 8.98755 \(\times\) 10^9 N \(\cdot\) m^2/C^2.

Your answer must be within ±1.0%

Correct answer: 66.0109 N/C.

Explanation:

Let:

\[ q = 5 \mu\text{C}, \]
\[ (x_1, y_1) = (0 \text{ m}, 49 \text{ m}), \]
\[ (x_2, y_2) = (27 \text{ m}, 0 \text{ m}), \]
\[ (x_3, y_3) = (29 \text{ m}, -66 \text{ m}). \]

\[
E_i \equiv \| \vec{E}_i \| = k \frac{Q}{r_i^2}
\]
gives the magnitude of the electric field due to the \(i^{th}\) charge, so
\[ \| \vec{E}_1 \| = k_e \frac{Q}{r_1^2} = k_e \frac{Q}{y_1^2} = (8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{5 \times 10^{-6} \text{ C}}{(49 \text{ m})^2} = 18.7163 \text{ N/C}, \]

\[ \| \vec{E}_2 \| = k_e \frac{Q}{r_2^2} = k_e \frac{Q}{x_2^2} = (8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{5 \times 10^{-6} \text{ C}}{(27 \text{ m})^2} = 61.643 \text{ N/C}, \]

and

\[ \| \vec{E}_3 \| = k_e \frac{Q}{r_3^2} = k_e \frac{Q}{x_3^2 + y_3^2} = (8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{5 \times 10^{-6} \text{ C}}{(29 \text{ m})^2 + (-66 \text{ m})^2} = 8.64686 \text{ N/C}. \]

The \( x \) and \( y \) components are

\[ E_{x,i} = \| \vec{E}_i \| \left( \frac{x_i}{r_i} \right) \] and \( E_{y,i} = \| \vec{E}_i \| \left( \frac{y_i}{r_i} \right). \]

\[ E_{x,1} = 0 \text{ N/C}, \]
\[ E_{y,1} = 18.7163 \text{ N/C}, \]
\[ E_{x,2} = 61.643 \text{ N/C}, \]
\[ E_{y,2} = 0 \text{ N/C}, \]
\[ E_{x,3} = \| \vec{E}_3 \| \left( \frac{x_3}{r_3} \right) = 8.64686 \text{ N/C} \left( \frac{29 \text{ m}}{72.0902 \text{ m}} \right) = 3.47841 \text{ N/C}, \]

and

\[ E_{y,3} = \| \vec{E}_3 \| \left( \frac{y_3}{r_3} \right) = 8.64686 \text{ N/C} \left( \frac{-66 \text{ m}}{72.0902 \text{ m}} \right) = -7.91637 \text{ N/C}, \]

so

\[ E_x = \sum_{i=1}^{3} E_{x,i} = E_{x,1} + E_{x,2} + E_{x,3} = 0 \text{ N/C} + 61.643 \text{ N/C} + 3.47841 \text{ N/C} = 65.1214 \text{ N/C}. \]

The net electric field is

\[ E_y = \sum_{i=1}^{3} E_{y,i} = E_{y,1} + E_{y,2} + E_{y,3} = 18.7163 \text{ N/C} + 0 \text{ N/C} - 7.91637 \text{ N/C} = 10.7999 \text{ N/C}, \]

so the magnitude of the net electric field is

\[ \| \vec{E} \| = \sqrt{E_x^2 + E_y^2} = \sqrt{(65.1214 \text{ N/C})^2 + (10.7999 \text{ N/C})^2} = 66.0109 \text{ N/C}. \]

**Electric Field 01 003 10.0 points**

Two charges are located in the \((x, y)\) plane as shown. The fields produced by these charges are observed at a point \(p\) with coordinates \((0, 0)\).

\[
\begin{array}{c}
-p \\
\text{3 m} \\
\text{3 m} \\
\text{7 C} \\
\text{1.6 m} \\
\text{1.8 m} \\
\text{8.1 C}
\end{array}
\]

Find the \(x\)-component of the electric field at \(p\). The value of the Coulomb constant is \(8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\).

Your answer must be within ± 1.0%.

Correct answer: \(-9.90208 \times 10^9 \text{ N/C}\).

**Explanation:**

Let: \((x_p, y_p) = (0, 0)\),
\((x_1, y_1) = (3 \text{ m}, -1.6 \text{ m})\),
\((x_2, y_2) = (-3 \text{ m}, -1.8 \text{ m})\),
\(q_1 = 7 \text{ C}\),
\(q_2 = -8.1 \text{ C}\), and
\(k_e = 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\).
Consider the electric field vectors:

\[ E_1 \]

\[ E_2 \]

\[ q_1 \]

\[ q_2 \]

where

\[ \theta_1 = 180^\circ - \tan^{-1} \left( \frac{1.6 \text{ m}}{3 \text{ m}} \right) = 151.928^\circ, \]

\[ \theta_2 = 180^\circ + \tan^{-1} \left( \frac{1.8 \text{ m}}{-3 \text{ m}} \right) = 210.964^\circ. \]

In the \( x \)-direction, the contributions from the two charges are

\[ E_{x1} = -k_e \frac{Q_1}{r_1^2} \cos \theta_1 = -k_e \frac{Q_1}{r_1^2} \frac{x_1}{r_1} \]

\[ = - \left( 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \]

\[ \times \frac{7 \text{ C}}{(3.4 \text{ m})^2} \frac{3 \text{ m}}{3.4 \text{ m}} \]

\[ = -4.80202 \times 10^9 \text{ N/C} \]

and

\[ E_{x2} = -k_e \frac{Q_2}{r_2^2} \cos \theta_2 = -k_e \frac{Q_2}{r_2^2} \frac{x_2}{r_2} \]

\[ = - \left( 8.98755 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \right) \]

\[ \times \frac{-8.1 \text{ C}}{(3.49857 \text{ m})^2} \frac{-3 \text{ m}}{3.49857 \text{ m}} \]

\[ = -5.10006 \times 10^9 \text{ N/C}, \] so

\[ E_x = E_{x1} + E_{x2} \]

\[ = -4.80202 \times 10^9 \text{ N/C} \]

\[ + (-5.10006 \times 10^9 \text{ N/C}) \]

\[ = -9.90208 \times 10^9 \text{ N/C}. \]
What is the equivalent capacitance for this network?
Your answer must be within ±1.0%
Correct answer: 6.5556 µF.
Explanation:

Let: \( C_1 = 2 \ \mu \text{F}, \)
\( C_2 = 5 \ \mu \text{F}, \)
\( C_3 = 2 \ \mu \text{F}, \)
\( C_4 = 5 \ \mu \text{F}, \) and
\( \mathcal{E}_B = 140 \ \text{V}. \)

\( C_1 \) and \( C_2 \) are connected in parallel, so
\( C_{12} = C_1 + C_2 = 7 \ \mu \text{F}. \)

\( C_{12} \) and \( C_3 \) are connected in series, so

\[
\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{C_3 + C_{12}}{C_{12} C_3}
\]
\[
C_{123} = \frac{C_{12} C_3}{C_3 + C_{12}} = \frac{(7 \ \mu \text{F})(2 \ \mu \text{F})}{7 \ \mu \text{F} + 2 \ \mu \text{F}} = 1.55556 \ \mu \text{F}. \]

\( C_{123} \) and \( C_4 \) are connected in parallel, so

\[
C = C_{4+123} = 5 \ \mu \text{F} + 1.55556 \ \mu \text{F} = 6.55556 \ \mu \text{F}. \]

Energy in a Capacitor

008 (part 1 of 2) 10.0 points
A 40 µF air-filled capacitor is charged to a potential difference of 5760 V.
What is the energy stored in it?
Your answer must be within ±1.0%
Correct answer: 663.552 J.

Explanation:

Let: \( C = 40 \ \mu \text{F} = 4 \times 10^{-5} \ \text{F} \) and
\( V = 5760 \ \text{V} \).

The energy stored in the capacitor is

\[
U = \frac{1}{2} C V^2 = \frac{1}{2} (4 \times 10^{-5} \ \text{F}) (5760 \ \text{V})^2 = 663.552 \ \text{J}. \]

009 (part 2 of 2) 10.0 points
If the voltage source remains connected to the capacitor, but the air is replaced by a layer of plastic with dielectric constant of 4.4, what is the new value of energy stored in it?
Your answer must be within ±1.0%
Correct answer: 2919.63 J.

Explanation:

Let: \( \kappa = 4.4 \).
\[ U = \frac{1}{2} \kappa C V^2 \]
\[ = \frac{1}{2} (4.4)(4 \times 10^{-5} \text{ F})(5760 \text{ V})^2 \]
\[ = 2919.63 \text{ J}. \]

Serway CP 16 65
010 10.0 points

Consider a parallel-plate capacitor with charge \( Q \) and area \( A \), filled with dielectric material having dielectric constant \( \kappa \). It can be shown that the magnitude of the attractive force exerted by each plate on the other is given by \( F = \frac{Q^2}{2 \kappa \varepsilon_0 A} \).

When a potential difference of 80 V exists between the plates of an air-filled 25 \( \mu \text{F} \) parallel-plate capacitor, what force does each plate exert on the other if they are separated by 2.8 mm?

Your answer must be within \( \pm 1.0\% \)

Correct answer: 28.5714 N.

Explanation:

Let: \( V = 80 \text{ V} \),
\( C = 25 \ \mu \text{F} = 2.5 \times 10^{-5} \text{ F} \), \( \text{and} \)
\( d = 2.8 \text{ mm} = 0.0028 \text{ m} \).

The magnitude of charge on each plate is \( Q = CV \) and the area of each plate is

\[ C = \kappa \varepsilon_0 \frac{A}{d} \]
\[ \kappa \varepsilon_0 A = C d, \]

so the force exerted on each plate is

\[ F = \frac{Q^2}{2 \kappa \varepsilon_0 A} = \frac{(C V)^2}{2 C d} = \frac{C V^2}{2 d} \]
\[ = \frac{(2.5 \times 10^{-5} \text{ F})(80 \text{ V})^2}{2 (0.0028 \text{ m})} \]
\[ = 28.5714 \text{ N}. \]