This print-out should have 11 questions. Multiple-choice questions may continue on the next column or page – find all choices before answering.

**Angular Position of a Wheel**

001 10.0 points

At \( t = 0 \), a wheel rotating about a fixed axis at a constant angular deceleration of 0.25 rad/s\(^2\) has an angular velocity of 1.3 rad/s and an angular position of 9.7 rad.

What is the angular position of the wheel after 4 s?

Correct answer: 12.9 rad.

**Explanation:**

Let : \( \alpha = -0.25 \text{ rad/s}^2 \),
\( \omega_0 = 1.3 \text{ rad/s} \),
\( \theta_0 = 9.7 \text{ rad} \), and
\( t = 4 \text{ s} \).

The angular position is
\[
\theta_f = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2 \\
= 9.7 \text{ rad} + (1.3 \text{ rad/s}) (4 \text{ s}) \\
+ \frac{1}{2} (-0.25 \text{ rad/s}^2) (4 \text{ s})^2 \\
= 12.9 \text{ rad}.
\]

**Balancing Tires**

002 10.0 points

A tire placed on a balancing machine in a service station starts from rest and turns through 5.81 rev in 1.67 s before reaching its final angular speed.

Find its angular acceleration.

Correct answer: 26.179 rad/s\(^2\).

**Explanation:**

Let : \( \omega_0 = 0 \text{ rev/s} \),
\( \theta = 5.81 \text{ rev} \), and
\( t = 1.67 \text{ s} \).

There is a constant angular acceleration, so
\[
\theta = \omega_0 t + \frac{1}{2} \alpha t^2 = \frac{1}{2} \alpha t^2 \\
\alpha = \frac{2 \theta}{t^2} = \frac{2 (5.81 \text{ rev})}{(1.67 \text{ s})^2} \cdot \frac{2 \pi \text{ rad}}{\text{rev}} \\
= 26.179 \text{ rad/s}^2.
\]

**Two Blocks**

003 (part 1 of 3) 10.0 points

The blocks shown in figure are connected by an inextensible string of negligible mass. The string passes over a frictionless pulley without slipping on the rim of the pulley. The block on the frictionless incline is moving with a constant acceleration up the incline.

Determine the tension in the vertical part of the string. The acceleration of gravity is 9.8 m/s\(^2\).

Correct answer: 75.6 N.

**Explanation:**

Let : \( a = 6.2 \text{ m/s}^2 \),
\( m_1 = 21 \text{ kg} \) and
\( g = 9.8 \text{ m/s}^2 \).

Considering a free-body diagram for the hanging mass \( m_1 \),
\[
m_1 g - T_1 = m_1 a
\]
\[
T_1 = m_1 (g - a) = (21 \text{ kg}) (9.8 \text{ m/s}^2 - 6.2 \text{ m/s}^2) = 75.6 \text{ N}.
\]
004 (part 2 of 3) 10.0 points
Determine the tension in the string parallel to the inclined plane.

Correct answer: 56.0644 N.

Explanation:

Let:

\[ R = 0.29 \text{ m}, \]
\[ \theta = 34^\circ, \]
\[ m_2 = 4.8 \text{ kg}, \]

\[ T_2 = m_2 \left( a + g \sin \theta \right) \]
\[ = (4.8 \text{ kg}) \left[ 6.2 \text{ m/s}^2 \right. \]
\[ + (9.8 \text{ m/s}^2) \sin 34^\circ \]
\[ = 56.0644 \text{ N}. \]

005 (part 3 of 3) 10.0 points
Find the mass of the pulley. It has uniform density and is shaped like a narrow cylindrical disk.

Correct answer: 6.3018 kg.

Explanation:

Applying torque to the pulley,

\[ (T_1 - T_2) R = I \alpha = I \left( \frac{a}{R} \right) \]

\[ I = \frac{(T_1 - T_2) R^2}{a} \]
\[ = \frac{(75.6 \text{ N} - 56.0644 \text{ N}) (0.29 \text{ m})^2}{6.2 \text{ m/s}^2} \]
\[ = 0.264991 \text{ kg m}^2. \]

For the disk

\[ I_{disk} = \frac{1}{2} M R^2 \]
\[ M = \frac{2 I}{R^2} = \frac{2 (0.264991 \text{ kg m}^2)}{(0.29 \text{ m})^2} \]
\[ = 6.3018 \text{ kg}. \]

Worker on Scaffold 006 10.0 points
A worker is standing on a scaffold supported by a vertical rope at each end. The scaffold weighs 210 N and is 2.68 m long.

What is the tension in the rope nearer the 674 N worker when he stands 0.551 m from one end? The acceleration of gravity is 9.8 m/s².

Correct answer: 640.428 N.

Explanation:

Let:

\[ W_1 = 210 \text{ N}, \]
\[ W_2 = 674 \text{ N}, \]
\[ \ell = 2.68 \text{ m} \quad \text{and} \quad x = 0.551 \text{ m}. \]

Set the pivot point at the end of the scaffold which is farthest from the worker. Because the system is in equilibrium, the total torque is equal to zero.

\[ F \ell - W_2 (\ell - x) - W_1 \left( \frac{\ell}{2} \right) = 0 \]

\[ F = W_2 \left( 1 - \frac{x}{\ell} \right) + \frac{W_1}{2} \]
\[ = (674 \text{ N}) \left[ 1 - \frac{0.551 \text{ m}}{2.68 \text{ m}} \right] + \frac{210 \text{ N}}{2} \]
\[ = 640.428 \text{ N}. \]
007 10.0 points
A circular-shaped object of mass 9 kg has an inner radius of 8 cm and an outer radius of 23 cm. Three forces (acting perpendicular to the axis of rotation) of magnitudes 12 N, 24 N, and 15 N act on the object, as shown. The force of magnitude 24 N acts 40° below the horizontal.

Find the magnitude of the net torque on the wheel about the axle through the center of the object.

Correct answer: 4.29 N · m.
Explanation:
Let :  
\[ a = 8 \text{ cm} = 0.08 \text{ m} , \]
\[ b = 23 \text{ cm} = 0.23 \text{ m} , \]
\[ F_1 = 12 \text{ N} , \]
\[ F_2 = 24 \text{ N} , \]
\[ F_3 = 15 \text{ N} , \text{ and} \]
\[ \theta = 40^\circ . \]

The total torque is
\[ \tau = a F_2 - b F_1 - b F_3 \]
\[ = (0.08 \text{ m})(24 \text{ N}) \]
\[ - (0.23 \text{ m})(12 \text{ N} + 15 \text{ N}) \]
\[ = -4.29 \text{ N} \cdot \text{ m} , \]
with a magnitude of 4.29 N · m.

---

Solid Sphere on an Incline 008 (part 1 of 2) 10.0 points
A solid sphere of radius 31 cm is positioned at the top of an incline that makes 21° angle with the horizontal. This initial position of the sphere is a vertical distance 3.4 m above its position when at the bottom of the incline. The sphere is released and moves down the incline.

Find the speed of the sphere when it reaches the bottom of the incline if it rolls without slipping. The acceleration of gravity is 9.8 m/s² and the moment of inertia of a sphere with respect to an axis through its center is \( \frac{2}{5} MR^2 \).

Correct answer: 6.89928 m/s.
Explanation:
From conservation of energy
\[ U_i = K_{trans,f} + K_{rot,f} \]
\[ M g h = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2 \]
\[ = \frac{1}{2} M v^2 + \frac{1}{2} \left( \frac{2}{5} M R^2 \right) \left( \frac{v^2}{R^2} \right) \]
\[ = \frac{7}{10} M v^2 \]
\[ v_1 = \sqrt{\frac{10}{7} g h} = \sqrt{\frac{10}{7} (9.8 \text{ m/s}^2)(3.4 \text{ m})} \]
\[ = 6.89928 \text{ m/s} . \]

009 (part 2 of 2) 10.0 points
Find the speed of the sphere if it reaches the bottom of the incline by slipping frictionlessly without rolling.

Correct answer: 8.16333 m/s.
Explanation:
From conservation of energy

\[ U_i = K_{\text{trans},f} \]
\[ M g h = \frac{1}{2} M v^2 \]
\[ v_2 = \sqrt{2 g h} = \sqrt{2(9.8 \text{ m/s}^2)(3.4 \text{ m})} \]
\[ = 8.16333 \text{ m/s}. \]

**Watermelon on Scaffold**

010 (part 1 of 2) 10.0 points

An 8.6 kg watermelon is placed at one end of a 7.6 m, 207 N scaffolding supported by two cables. One supporting cable is at the opposite end of the scaffolding, and the other is 0.93 m from the watermelon.

How much tension is in the cable at the end of the scaffolding? The acceleration of gravity is 9.8 m/s\(^2\).

Correct answer: 77.3178 N.

**Explanation:**

Let:
\[ m = 8.6 \text{ kg} \]
\[ W = 207 \text{ N}, \]
\[ \ell = 7.6 \text{ m}, \]
\[ x = 0.93 \text{ m}, \text{ and} \]
\[ g = 9.8 \text{ m/s}^2. \]

Let the fulcrum be at the point of attachment of the cable closest to the watermelon.

The watermelon \(mg\) acts down (CCW) at a distance \(x\) from the fulcrum. The weight \(W\) acts down (CCW) at a distance \(\ell - x\) from the fulcrum, and the rightmost tension \(T_1\) acts up (CCW) at a distance \(\ell - x\) from the fulcrum, SO

\[ \sum \tau = \sum \tau_{\text{CW}} - \sum \tau_{\text{CCW}} = 0 \]
\[ W(\frac{\ell}{2} - x) - m g x - T_1 (\ell - x) = 0. \]
\[ W(\ell - 2 x) - 2 m g x = 2 T_1 (\ell - x) \]
\[ T_1 = \frac{W(\ell - 2 x) - 2 m g x}{2(\ell - x)} \]
\[ = \frac{(207 \text{ N})[7.6 \text{ m} - 2(0.93 \text{ m})]}{2(7.6 \text{ m} - 0.93 \text{ m})} \]
\[ - \frac{2(8.6 \text{ kg})(9.8 \text{ m/s}^2)(0.93 \text{ m})}{2(7.6 \text{ m} - 0.93 \text{ m})} \]
\[ = 77.3178 \text{ N}. \]

011 (part 2 of 2) 10.0 points

How much tension is in the cable closest to the watermelon?

Correct answer: 213.962 N.

**Explanation:**

Let the fulcrum be at the point of attachment of the rightmost cable.

The watermelon \(mg\) acts down (CCW) at a distance \(\ell\) from the fulcrum. The weight \(W\) acts down (CCW) at a distance \(\frac{\ell}{2}\) from the fulcrum, and the tension \(T_2\) acts up (CCW) at a distance \(\ell - x\) from the fulcrum. Thus

\[ T_2 (\ell - x) - m g \ell - W \left( \frac{\ell}{2} \right) = 0. \]

Multiplying by 2,
\[ 2 T_2 (\ell - x) = 2 m g \ell + W \ell \]
\[ T_2 = \frac{2 m g \ell + W \ell}{2(\ell - x)} \]
\begin{align*}
&= \frac{2 (8.6 \text{ kg}) (9.8 \text{ m/s}^2) (7.6 \text{ m})}{2 (7.6 \text{ m} - 0.93 \text{ m})} \\
&\quad + \frac{(207 \text{ N}) (7.6 \text{ m})}{2 (7.6 \text{ m} - 0.93 \text{ m})} \\
&= \boxed{213.962 \text{ N}}.
\end{align*}